Phase noise analysis in CMOS LC Quadrature VCO

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Abstract: Based on a physical understanding of phase noise mechanisms in CMOS LC Quadrature voltage-controlled oscillator (QVCO), a quantitative analysis of phase noise in QVCO is developed. The noise of the coupling transistor is identified, and the relationship between the phase noise and the ratio of the width of coupling transistor to the width of switching transistor is analyzed. The prediction of the phase noise is in good agreement with the measurement results at $1/f^2$ region. As a demonstration, a QVCO is implemented in a 0.25-μm CMOS process using on-chip spiral inductors. Measurement results show that the QVCO can be tuned between 1.77 and 2.415GHz, has a phase noise of $-117.17$dBc/Hz at 1-MHz frequency offset for a 2.267GHz carrier, and dissipates 7.5mW from the 2.5-V power supply.

Key Words: CMOS, phase noise, quadrature, thermal noise, radio frequency (RF), QVCO.

I. INTRODUCTION

Over the years, several phase noise analyses have been developed using the linear time-invariant or the time-variant models [1]-[5], all these studies of phase noise are based on a single LC oscillator. However, in quadrature LC oscillators, in addition to the phase noise of single LC oscillator, some extra phase noise which caused by the coupling transistor is introduced. Because in QVCO, two symmetric LC-tank oscillators are combined together by the coupling transistors to force two oscillators to oscillate in quadrature [6], in certain durations within one period, the coupling and switching transistors are all on and thus contribute to the phase noise. So compared with the single oscillator, the phase noise of quadrature oscillator is expected worse.

In this paper, we focus on phase noise analysis caused by the coupling transistors. One simple model is introduced to analyze the processes that convert thermal noise of the coupling transistors into phase noise of QVCO. The relationship between phase noise and the ratio of the width of coupling transistor to the width of
switching transistor is identified, and a simple quantitative expression accurately specifies thermally induced phase noise in QVCO. A good agreement is found between theory computation and the measurement results at $1/f^2$ region.

II. THERMALLY INDUCED PHASE NOISE

As shown in Fig 1, a single LC oscillator includes a LC tank (the shunt resistance $R$ represents the resistance loss of inductor $L$) and an active element. The active element has the remarkable property that it presents negative resistance to compensate for the dissipation by the lossy LC tank. When an overall negative resistance precisely cancels with the shunt resistance of the LC tank, the oscillation frequency $\omega$ is equal to the tank resonance frequency $\omega_0 = 1/\sqrt{LC}$.

![Fig.1: The model of LC oscillator](image)

When the oscillation frequency $\omega$ displaces $\Delta \omega$ from $\omega_0$, the transfer function $H(\omega_0 + \Delta \omega)$ of the LC tank can be expressed by:

$$H(\omega_0 + \Delta \omega) = \frac{j(\omega_0 + \Delta \omega)L}{1 - (\omega_0 + \Delta \omega)^2 LC}$$  \hspace{1cm} (1)

In a parallel RLC tank, the quality factor of the tank $Q = R/\omega_0 L = 1/\omega_0 L G$, $G$ is the reciprocal of the tank resistance $R$. When $\Delta \omega \ll \omega_0$, $H(\omega_0 + \Delta \omega)$ may be approximated by:

$$H(\omega_0 + \Delta \omega) \approx \frac{j\omega_0 L}{2Q\Delta \omega} \frac{1}{G}$$  \hspace{1cm} (2)

(1): Resonator Noise

In a RLC resonator, the only noise source is the thermal noise of the tank resistance $R$. Multiplying the noise current power spectral density (PSD) of the tank resistance $R$ by the squared transfer function of the parallel RLC tank to obtain the PSD of the noise voltage:
The single sideband phase noise is described by the ratio of the PSD of the noise voltage at a given frequency $\omega_0 + \Delta \omega$ to the mean-square carrier voltage at the carrier frequency $\omega_0$. Thus, the phase noise due to the resistance loss of RLC resonator is:

$$L_x(\Delta \omega) = 8kT \frac{1}{V_s^2} \frac{1}{G} \left( \frac{\omega_0}{2Q\Delta \omega} \right)^2$$

(4)

Where $V_s$ is the amplitude of the carrier signal.

(2): The phase noise of the coupling transistors

In a QVCO, the coupling transistors force two identical LC-tank VCOs to oscillate in quadrature. The schematic of a simple QVCO is shown in Fig. 2, where the coupling between two VCOs is performed by transistors $M_{cp}$ and placed in parallel with the switching transistors $M_{sw}$. Under steady-state oscillation, the quadrature signals will be produced at the QVCO outputs.

Now, Looking at the left half-oscillator of the QVCO in Fig. 2, because the phase of $Q_+$ lags/leads $90^\circ$ behind/to the phase of $I_+$, $M_{cp1}$ & $M_{sw2}$ or $M_{sw1}$ & $M_{cp2}$ are all on in the quarter of one oscillation period, the thermal noise of these transistors will be injected into the LC-tank and be converted into phase noise of the QVCO.

To gain more insight into the phase noise effect of the coupling transistors, the operation of QVCO is divided into four phases in one oscillation period. The working mechanism and oscillation waveform of the QVCO are shown in Fig. 3. During phase 1 and phase 3, $M_{sw1}$ & $M_{cp2}$ and $M_{cp1}$ & $M_{sw2}$ are on respectively,
the current noise of the coupling and switching transistors will be injected into LC-tank, while during phase 2
and phase 4, the transistors will be degenerated and their current noise is rejected.

![Diagram of QVCO working mechanism and oscillation waveform.]

During phase 1, $M_{sw1}$ & $M_{cp1}$ are all on and their operation is identical to that of a differential pair. The
thermal noise model is shown in Fig. 4. $\overline{i_{sw1}}$ & $\overline{i_{cp1}}$ are the channel thermal noise current of transistors
$(\overline{i_{sw1}} = 4kT\gamma g_{m1}, \overline{i_{cp1}} = 4kT\gamma g_{m2})$ [7], where $g_{m1}$ and $g_{m2}$ denote the transconductance of the switching
and coupling transistors respectively.

![Diagram of the noise model during phase 1.]

Neglecting the channel-length modulation, the total noise current $i_{ax}$ caused by $\overline{i_{sw1}}$ & $\overline{i_{cp1}}$ can be expressed
as follows:

$$i_{ax} = i_{sw1} - i_{cp2} = 2\left(\frac{g_{m1}}{g_{m2} + g_{m1}} \cdot \overline{i_{sw1}} + \frac{g_{m2}}{g_{m2} + g_{m1}} \cdot \overline{i_{cp1}}\right)$$
Where \( i_{on1} \) and \( i_{on2} \) denote the noise current at node \( I_+ \) and \( I_- \) respectively.

Since \( \overline{i_{on1}} \) and \( \overline{i_{on2}} \) are uncorrelated, then the total mean-square noise current \( \overline{i_{on}^2}(f,t) \) is:

\[
\overline{i_{on}^2}(f,t) = 16kTg_{s_w1}g_{s_w2}^2 \gamma G(t)
\]

Assuming \( I_+ = V_o \sin(\omega t) \), \( Q_- = V_o \cos(\omega t) \), the coupling coefficient \( \alpha \) is defined as the ratio of the width \( W_{cp2} \) of the coupling transistor \( M_{cp2} \) to the width \( W_{sw1} \) of the switching transistor \( M_{sw1} \). Hence, the transconductance of \( M_{sw1} \) & \( M_{cp2} \) can be approximated by

\[
g_{s_w1} = K \sin(\omega t) \quad \text{and} \quad g_{s_w2} = \alpha K \cos(\omega t)
\]

Then \( G(t) \) can be rewritten as:

\[
G(t) = \frac{\alpha KV_o}{\sqrt{1 + \alpha^2}} \cdot \frac{\sin(2\omega t)}{\sin(\omega t + \varphi)} \quad \text{(\( \varphi = \arcsin(\frac{\alpha}{\sqrt{1 + \alpha^2}})) \}
\]

The time-average noise current PSD due to the coupling transistor is given by:

\[
\overline{i_{on}^2}(f) = 8kTg_\gamma \left( \frac{1}{T} \right)^2 \int_0^T G(t)dt = 8kTg_\gamma G(t)
\]

Where \( T \) is the period of oscillation signal.

Through a series of complex derivation and simplicity, equation (8) can be expressed as:

\[
\overline{i_{on}^2}(f) = 4kTg_\gamma \cdot \frac{\alpha KV_o}{\pi \sqrt{1 + \alpha^2}} \left[ 2 \cos(\varphi) + 2 \sin(\varphi) \cdot \ln \left( \frac{\tan(\frac{\varphi}{4} + \frac{\theta}{2})}{\tan(\frac{\theta}{2})} \right) \right]
\]

During phase 3, the noise current PSD is equal to \( \overline{i_{on}^2}(f) \). So the total noise current contribution \( \overline{i_{total}^2} = 2\overline{i_{on}^2}(f) \). Then the total noise voltage \( \overline{v_{total}^2} \) is equal to \( \overline{i_{total}^2} \cdot \left| H(\omega_0 + \Delta\omega) \right|^2 \). Hence, the total phase noise contribution \( L_{cp}(\omega) \) which induced by the coupling transistor can be proved that:

\[
L_{cp}(\omega) = 16kTg_\gamma \cdot \frac{\alpha KV_o}{\pi \sqrt{1 + \alpha^2}} \cdot \left[ 2 \cos(\varphi) + 2 \sin(\varphi) \cdot \ln \left( \frac{\tan(\frac{\varphi}{4} + \frac{\theta}{2})}{\tan(\frac{\theta}{2})} \right) \right]
\]

(3): The noise of the switching transistors and the tail current noise

Let us look at the left half VCO in Fig.2, when \( M_{sw1} \) or \( M_{sw2} \) is off, the switching pair does not contribute to the phase noise, however, when \( M_{sw1} \) and \( M_{sw2} \) are all on during the part of the oscillation period, the thermal noise of the switching pair will be injected into LC-tank. The phase noise analysis of the switching
transistors is similar to the phase noise analysis of the coupling transistors.

Assuming during the time internal $\Delta$, both $M_{sw1}$ and $M_{sw2}$ are all on and $V_s$ and $-V_s$ are the corresponding oscillation amplitude (as shown in Fig.5). During time internal $\Delta$, the total transconductance $G(t) = 2 \cdot (g_{sw1} \cdot g_{sw2} / (g_{sw1} + g_{sw2}))$, then the time-average current noise PSD $\overline{i_{s\text{noi}}^2}$ is:

$$\overline{i_{s\text{noi}}^2} = 8kT\gamma(\int_0^\Delta G(t) \cdot dt) = 8kT\gamma \cdot G(t)$$

(11)

$G(t)$ can be calculated as [6]. Then $\overline{i_{s\text{noi}}^2}$ can be approximated by

$$\overline{i_{s\text{noi}}^2} = \frac{16kT\gamma}{\pi} \frac{I_{bias}}{V_o}$$

(12)

Where $I_{bias}$ is the tail current due to $V_{bias}$.

Then the phase noise $L_{sw}(\omega)$ that caused by the switching pair can be written as:

$$L_{sw}(\omega) = \frac{32kT\gamma}{\pi} \frac{I_{bias}}{V_o} \cdot R^2 \cdot \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2$$

(13)

In [5], the phase noise mechanisms caused by the tail current noise is analyzed. The phase noise contribution $L_{sw}(\omega)$ due to the tail current noise is:

$$L_{sw}(\omega) = \frac{32}{9} \frac{kT}{g_{sw1}} \frac{kTR}{V_e^2} \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2$$

(14)

Therefore, in the QVCO, summing equation (4), (10), (13) and (14) for thermally induced phase noise from the resonator, switching pair, coupling transistors and tail current respectively, the total thermally induced phase noise $L(\omega)$ can be expressed as

$$L(\omega) = L_x(\omega) + L_{sw}(\omega) + L_{sw}(\omega) + L_{sw}(\omega) = \frac{4FkTR}{V_e^2} \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2$$

(15)

It has the same form with the Leeson’s Hypothesis. But in Leeson’s equation, $F$ is an unspecified factor. In our analysis, $F$ is a specified form, which is expressed as:
\[
F = 2 + \frac{8\gamma BV}{\pi V_c} + \frac{8}{9} \gamma g_m R + \frac{4\pi \alpha KV R}{\pi \sqrt{1 + \alpha^2}} \left[ 2 \cos(\phi) + 2 \sin(\phi) - \sin(2\phi) \left( \ln \frac{\tan(\phi/2)}{\tan(\phi/2)} \right) \right]
\]

(where \( \phi = \arcsin \left( \frac{\alpha}{\sqrt{1 + \alpha^2}} \right) \) and \( \alpha = \frac{W_{cm}}{W_{ml}} \)).

From the expression of phase noise \( L(\omega) \), we notice that phase noise varies with the coupling coefficient \( \alpha \).

Defining \( L_\alpha(\omega) = L(\omega) \bigg| \omega = \omega_0 \), \( L(\omega) \) denotes the phase noise contribution as a function of \( \alpha \), the computation result of \( L_\alpha(\omega) \) is demonstrated in Fig.6. Clearly, the phase noise contribution gets larger when the coupling between two VCOs is enhanced by increasing the coupling coefficient \( \alpha \). But quadrature phase error will decrease with \( \alpha \) increasing [9]. As shown in the above analysis, phase noise and phase error are conflicting parameters in the QVCO. So in designing a QVCO, the optimal \( \alpha \) is selected based on requirements of the designing system.

![Fig.6: The phase noise variation with \( \alpha \)](image)

(The loaded quality factor \( Q = 7.4 \), \( \gamma = 3 \))

**III. QVCO DESIGN AND MEASUREMENT RESULTS**

A QVCO has been fabricated in a standard 0.25-um CMOS process with five layers. The core schematic of the QVCO is shown in Fig.7. The NMOS \( M_{out} - M_{out} \) constitute the cross-coupled pairs, which generate negative resistance to cancel the loss of the LC tank. The NMOS \( M_{op} - M_{op} \) are the coupling transistor and they force two VCOs to oscillate in quadrature. The widths of the MOS transistors are selected to achieve the coupling coefficient \( \alpha = 0.5 \) and the maximal output amplitude. In this design, the 3.5-turn on-chip inductor that offered by manufacturer is applied, each inductor is about 3.74nH, the quality factor \( Q \) is about 7.4 when
the oscillation frequency is about 2GHz. The die photograph of this design is shown in Fig.8. The chip area is about 1000um by 710um. The measured phase noise is about $-117.17 \text{dBc/Hz}$ at offset frequency 1MHz for the 2.267GHz oscillation frequency (as shown in Fig.9). The oscillator is tuned from 1.777GHz to 2.415GHz through the accumulation varactors which are offered by manufacturer. The total power consumption of the core circuit is only 7.5mW with 2.5V supply voltage. Certainly, increasing the power consumption will achieve the lower phase noise, as shown in the phase noise expression $L(\omega)$. Fig.9 depicts the excellent agreement between the measured phase noise and the predicted phase noise at region A. In region B and region C, there is larger difference between the measured phase noise and the predicted phase noise. The larger phase noise difference in region B is because of the flicker noise up-conversion, while in region C is caused by the noise of output buffer.

![Fig.7: The core schematic of QVCO](image1)

![Fig.8: Die photograph of QVCO](image2)

![Fig.9 The measured phase noise is compared with predictions from analysis](image3)

(The loaded quality factor $Q = 7.4$, the coupling coefficient $\alpha = 0.5$, $\gamma = 3$)
IV. CONCLUSIONS AND DISCUSSIONS

The contribution of this paper is to analyze the thermally induced phase noise due to coupling transistors, and an analytical expression has been derived, moreover, a complete phase noise expression is developed, and good agreement is found between theory computation and measured results at $1/f^2$ region.

In our expression of phase noise, since flicker noise has not been dealt with, the slope of phase noise is $-20\text{dBc/dec}$, if considering the flicker noise up-conversion, the slope of phase noise will be $-20\text{dBc/dec} \sim -30\text{dBc/dec}$, thus the difference between the theory computation and the measured phase noise becomes larger at small offset frequency.

A QVCO has been fabricated using 0.25-um CMOS process to verify the analysis. The measured phase noise is about $-117.17\text{dBc/Hz}$ at an offset frequency of 1MHz with the power consumption 7.5mW. The tuning range is from 1.777GHz to 2.415GHz.

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